

## **Volatility spillover effect from commodities to Brazilian stock markets in the period 2000–2016: is there a possibility of diversification?**

Pedro Raffy Vartanian<sup>1</sup>

**Resumo:** Em 2002 teve início um ciclo de alta nos preços das commodities no mercado internacional de forma concomitante à trajetória dos principais índices das bolsas de valores, com destaque para as bolsas dos países emergentes. No Brasil, especificamente, o principal índice de ações (Ibovespa) apresenta um histórico de composição importante de empresas ligadas ao setor de commodities, o que sugere uma relação entre o preço das commodities e o mercado acionário. Assim, o objetivo do estudo é avaliar o contágio de volatilidade entre o preço das commodities e o Ibovespa, por meio de um modelo GARCH Multivariado, com o objetivo de se verificar a possibilidade de diversificação de investimentos. A pesquisa tem como hipótese o fato de que há uma forte relação entre os preços das commodities e o índice Ibovespa, com a presença do efeito contágio de acordo com Forbes e Rigobon (2002), o que inibe a diversificação de investimentos entre as ações que compõem o índice e as commodities no cenário internacional. Os resultados encontrados corroboraram parcialmente a hipótese formulada, já que foi possível observar um forte aumento da covariância condicional entre as duas variáveis durante a crise financeira internacional. Por outro lado, a correlação condicional entre o Ibovespa e o preço das commodities mostrou que a relação entre as variáveis é relativamente baixa nos períodos anterior e posterior à crise de 2008, o que sugere que investimentos concomitantes em commodities e Ibovespa constituem uma estratégia de diversificação do risco, ao contrário do que comumente se supõe.

**Palavras-chave:** Contágio, Volatilidade, Commodities, Ibovespa, GARCH Multivariado

**Classificação JEL:** G11, G15, C58

**Abstract:** In 2002, a commodity prices boom in the international market began concomitantly with the trend of the main stock market index, especially in emerging stock exchanges. In Brazil, specifically, the main stock index (Ibovespa) presents a history of important composition of companies linked to the commodities sector, which suggests a relationship between the commodity prices and the stock market. Thus, the objective of the study was to evaluate the contagion of volatility between the commodity prices and the Ibovespa, through a multivariate GARCH model, to verify the possibility of diversification of investments. The research hypothesized that there is a strong relationship between the commodity prices and the Ibovespa index, with the presence of the contagion effect according to Forbes and Rigobon (2002), which inhibits the diversification of investments between the stocks that make up the index and the commodities on the international scene. The results partially corroborate the hypothesis formulated, since it was possible to observe a strong increase in conditional covariance between the two variables during the international financial crisis. On the other hand, the conditional correlation between the Ibovespa and the commodity prices showed that the relationship between the variables was relatively small in the periods before and after the 2008 crisis, which suggests that concomitant investments in commodities and the Ibovespa constitute a risk diversification strategy, contrary to what is commonly supposed.

**Keywords:** Contagion, Volatility, Commodities, Ibovespa, Multivariate GARCH.

**JEL Classification:** G11, G15, C58.

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## 1. Introduction

The increase in commodity prices in the international market since 2002, the sharp decline observed during the subprime crisis and its impact on other markets, such as the stock market and foreign exchange, constituted material facts in a highly integrated international financial system. Interestingly, similar behavior to that of commodity prices was observed in the main stock exchanges, especially exchanges in emerging countries. Changes in commodity prices often exert an impact on the financial system through three large groups of investments: futures, ETF (exchange-traded fund) from commodity companies and shares in companies linked to the sector. An additional group can be included, since, especially for developed countries, many studies, such as Kling (1985), have associated the effects on stock markets with the oil shocks in the 1970s and shown that the rise in oil prices may be associated with a decline in the stock market, given that the supply shock results, from the macro-economic viewpoint, from the combination of recession with inflation. In Brazil, specifically, the main index of the stock exchange's shares (Ibovespa) has a history of important composition of companies linked to the commodity sector, which suggests, in principle, an important relationship between the commodity prices and the behavior of the Ibovespa. Despite the importance of commodities in the economy and the trade balance composition, it can be said that in recent years, there was a significant reduction in participation of companies linked to the commodities sector in the Ibovespa composition.

The contagion relations between the exchanges of different countries' values have constituted a recurring theme of research in the last decade. The number of studies on the effects of US stock market behavior on various emerging countries has increased significantly. Some recent research has included in the assessment of the contagion effect the behavioral effects of oil prices and other commodities on the stock exchanges, mainly from oil-exporting countries. However, little research has investigated the Ibovespa's behavior in relation to commodity prices. In this sense, the objective of this study is to evaluate the volatility spillover of commodity prices on the international scene to the Ibovespa, through a multivariate GARCH model, to verify the possibility of diversification of investments between the commodities and the Brazilian stock index. The study is relevant because it allows considerations about the possibility of diversification (or not) between applications in commodities and shares in the Brazilian market. The research hypothesizes that there is a strong relationship between the commodity prices and the Ibovespa index, with the presence of the contagion effect, which inhibits the diversification of investments among the stocks that make up the index and the commodity prices in the international scenario.

Thus, the article is structured as follows. In addition to this introduction, section 2 presents the theoretical framework about the contagion effect and the transmission of volatility, especially for applications to financial and commodity markets. Additionally, the section includes a review of the literature on the behavior of commodity prices over time. In this context, despite being a major exporter of commodities, the Brazilian financial market does not appear in international studies, including those that sample mainly the oil-exporting countries. Section 3 initially presents a description of the data used in the survey and later the econometric methodology employed, which consists of using a multivariate GARCH model, which originated in studies of autoregressive conditional heteroskedasticity (the ARCH effect) in univariate models, from the seminal work of Engle (1982). Subsequently, the volatility univariate model was complemented with studies of the generalized models of autoregressive conditional heteroskedasticity (GARCH), presented by Bollerslev (1986). Multivariate volatility models developed following the work of Engle, Granger and Kraft (1984), and the initial advances of this econometric modeling strategy sought to address some issues related to

the overparameterized model and the consequent need for restrictions to be imposed in the conditional variance matrix with the aim of reducing the number of estimated parameters. Further, section 4 presents the analysis of the results and a discussion. Finally, section 5 contains the final considerations.

## **2. Theoretical Foundation**

The research is based theoretically on the contagion effect and transmission (spillover) volatility, especially research that has addressed the relationship between volatility variables related to commodity markets and stock exchanges. In general, the literature on the topic has addressed separately the contagion effect of so-called interdependence. Forbes and Rigobon (2002), for example, defined contagion as an increase in the connections between two markets when a country suffers some kind of shock. The infection is characterized, in that sense, as the expansion of connections after the crash. The fact that the two markets are historically connected is not characterized, by itself, as the contagion effect, but as the interdependence. According to Forbes and Rigobon (2012), the connection between the two markets can be assessed by the correlation coefficient. The contagion effect has often been approached from the standpoint of volatility overflow, known as volatility spillover. The first studies on the contagion effect used the stock market as a major field of study. The studies assessed the existence of the contagion effect to identify the possibility of international diversification of investments, following Park's (2010) approach. Based on these concepts, Marçal et al. (2011) evaluated the presence of the contagion effect or interdependence between countries through the behavior of the stock markets, using a multivariate GARCH model, and found that, in the analyzed period (1994–2003), a regional contagion effect was present in Latin American and Asian countries.

Overall, some research has sought answers to the questions related to the volatility in commodity prices and to the strong upward cycle that occurred from 2002 until the subprime crisis. Jacks, O'Rourke and Williamson (2011) evaluated the behavior of the volatility of commodity prices in a study that included a period of 300 years. In a historical analysis of the period 1700 to 2008, the authors discussed some stylized facts about the behavior of prices. Although the purpose of the study was to compare the volatility of commodity prices with the prices of manufactured goods and to assess the effects on the economic growth of the countries, the authors demonstrated that the volatility of the commodity prices did not increase over time. Nevertheless, the study pointed out that, throughout history, commodity prices have shown greater volatility than the prices of manufactured goods as a result of the Industrial Revolution. Finally, the study found that, in the 300 years of analysis, the effect of economic isolation and war on the volatility of commodity prices was greater than the effect of global integration and periods of absence of conflict. In other words, the authors found evidence that increased integration of markets is an important way to reduce the volatility of commodity prices.

As regards the increase observed from 2002 to the subprime crisis, Prates (2007) pointed out that this behavior was due to a combination of factors, such as the global economic recovery process, the devaluation of the dollar, the increase in global liquidity and the low interest rates in the US economy generating a speculative bubble in the commodity market, and the Chinese economic growth.

Specifically regarding the effects of commodities' prices on the stock market, various surveys, using different econometric methodologies, have evaluated the contagion effect or interdependence between the two markets. Arouri, Lahiani and Nguyen (2011) analyzed the volatility transmission in 2005–2010 between oil prices and stock indexes in six countries that make up the Gulf Cooperation Council (Oman, the United Arab Emirates, Saudi Arabia, Qatar, Bahrain and Kuwait). The authors used a vector autoregressive model (VAR) combined with a

multivariate GARCH model and found that only in three of the six countries was the stock market significantly influenced by the oil prices. Nevertheless, the authors identified the overflow of the volatility of oil prices and the stock market across the board.

In another study, Teulon and Guesmi (2014) also evaluated the effect of oil prices on the stock market of oil-exporting countries. The difference was that the authors used the stock market data from four countries (Venezuela, the United Arab Emirates, Saudi Arabia and Kuwait) plus a global stock market index to evaluate the effects of oil prices on these markets. The authors identified an increase in volatility in equity markets before an expansion of volatility in oil prices with a methodology similar to the that adopted by Arouri, Lahiani and Nguyen (2011). In addition, similar results were obtained by Ghorbel, Abbes Boujelbene and Boujelbene (2014) when evaluating the contagion effect of oil prices for 10 OECD countries' stock exchanges.

In another analysis, Basher and Sadorsky (2016) estimated a range of multivariate volatility to identify the effects of oil prices on the MSCI (Morgan Stanley Capital International) emerging markets. The authors extended the analysis to include the effects of gold prices, bond prices and the VIX (CBOE Volatility Index), which is a volatility index that is based on the options of the S&P 500. The analysis of the contagion effect between these markets was intended to find a great hedge rate between assets and distinct indices.

Specifically in relation to the Brazilian case, the investigation of the contagion effect of applications has been directed more to the evaluation of the effects of stock markets, such as the analysis by Righi and Ceretta (2013), which estimated the contagion effect of the stock markets in the US, Mexico, Argentina and China on the Ibovespa using a multivariate GARCH model. In the commodities market specifically, Margarido and Turolla (2014) evaluated the contagion effect of oil prices on soybeans. The authors applied a co-integration model to identify the presence of short-term and long-term effects and GARCH multivariate analysis of variance for the series. Among the results, the authors identified the absence of the influence of the volatility of oil prices and soybeans.

The study by Vartanian (2012) sought to evaluate the contagion effect of commodity prices for the Ibovespa and the effects of the US stock market and the exchange rate on the stock index in Brazil. Through the use of a VAR model and impulse response functions, the author found that increases in commodity prices have a positive impact on the Ibovespa. The other variables, the exchange rate and the Dow Jones, also influenced the Ibovespa during the period. However, the study did not assess the spillover of volatility between assets and/or markets. In this sense, the present study is apparently the precursor analysis of volatility contagion between the commodity prices in the international market and the Ibovespa.

### **3. Methodology**

#### **3.1 Data**

For the application of the econometric multivariate model, which will be presented in the next subsection, the research used the daily data of two variables. One of the variables is the Ibovespa index, collected on a daily basis in Economica and converted into US dollars from January 2000 to June 2016. The reason for the conversion into dollars was to match the variables used in the estimation. The Ibovespa was included in the compound model for the return, that is, with the logarithm of the difference, which is approximately the ratio of the rate of change. The other variable, the CRB Index (Commodity Research Bureau), is a leading indicator that reflects the prices of various commodities in the international market, such as oil, natural gas, grains, meat, coffee, cocoa and precious metals. The index was calculated on the basis of Bloomberg with the same periodicity as the Ibovespa and considered as an asset in the sense of the return on an investment in commodities. The international financial market cannot

be applied in any asset linked to the CRB, but there are a number of options that approximate the return of the index. The CRB index was included in the model by compound return in the same way as the Ibovespa. This means that the logarithmic transformation and differentiation resulted in a series containing the daily returns of the CRB as if it were an asset. The fact that the CRB contemplates commodity prices in the international market in dollars was the determining reason for using the Ibovespa in dollars as well. Thus, the analysis consisted of variables indexed in the same currency. In addition, it should be noted that on national holidays and unavailability data of the Ibovespa and/or CRB, an observation of the previous day is repeated.

The use of the CRB was due to the availability of data and the frequency (daily) in the search period, since other indicators did not show the same characteristics. Using the IC-Br, which is the Commodities Index of Brazil's Central Bank, on a monthly basis would be inadequate due to the reduced number of observations available to estimate the econometric model. Oil prices in the international market have daily data, but the fact that Brazil is not a major exporter of oil and has a diversified export basket of commodities makes the use of this indicator not as relevant as the use of the CRB. Thus, the CRB index proved to be a better indicator with the frequency required for the purposes of this study.

**Table 1.** Descriptive statistics of the daily variables in the period January 3, 2000–June 30, 2016

	<b>Ibovespa (R\$)</b>	<b>Ibovespa (US\$)</b>	<b>CRB</b>	<b>R_Ibovespa (R\$)</b>	<b>R_Ibovespa (US\$)</b>	<b>R_CRB</b>
Average	40818.42	19522.86	368.29	0.000259	0.000124	0.000139
Median	46510.00	17768.19	380.43	0.000000	0.000000	0.000000
Maximum	73516.00	44616.04	579.68	0.136794	0.196125	0.043604
Minimum	8370.000	2160.33	205.62	-0.120961	-0.182352	-0.050720
Standard deviation	19425.31	11923.20	101.81	0.017880	0.023115	0.004615
Asymmetry	-0.226890	0.349450	0.0001	-0.068103	-0.158362	-0.513147
Kurtosis	1.611594	1.853999	1.749731	6.982842	8.594808	12.03129
Coeff. of variation	0.475896	0.610730	0.276425	69.03474	186.41129	33.20143
Jarque–Bera	382.6235	323.1191	280.3297	2847.435	5630.152	14812.63
Probability	0.00	0.00	0.00	0.00	0.00	0.00
Observations	4304	4304	4304	4303	4303	4303

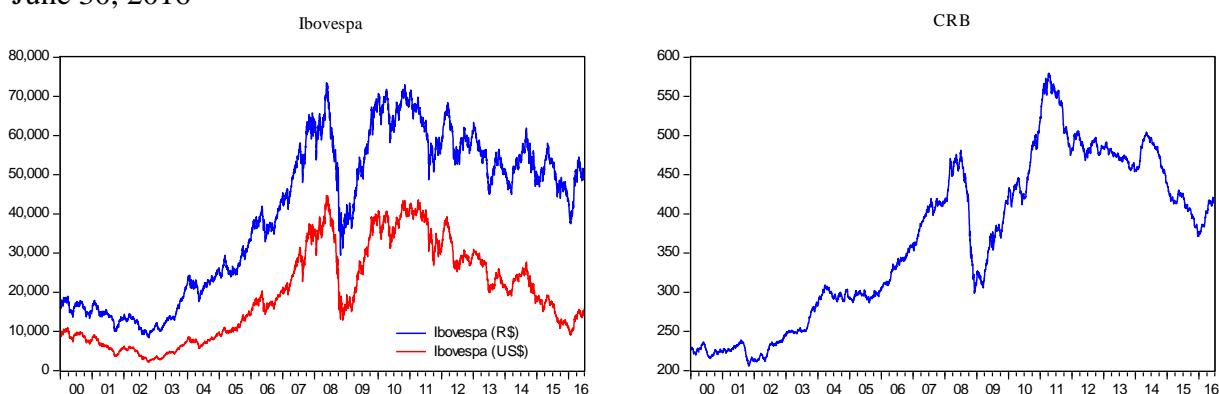
Source: Prepared from data obtained from Economática and Bloomberg.

Table 1 shows the descriptive statistics and the results of normality test of the variables used in the study. Although the estimation used the compound of the Ibovespa return in dollars and the CRB index, it was decided also to present the descriptive statistics of the variables in level, including the Ibovespa in Reais (local currency) in the first three columns. It is apparent that the Ibovespa average in Reais, which is 40818 points, is more than twice the Ibovespa average in dollars, which is 19522 points. Differences also appear in the median and in the maximum and minimum values. With respect to the asymmetry, it appears that the Ibovespa in real has negative asymmetry, while the same rate in dollars has positive skewness. A feature common to both is excess kurtosis, which is a feature present in the financial series.

Regarding the CRB, it is worth noting that the indicator is an index number (base 100 = 1960) and the relative volatility, measured by the coefficient of variation, is lower than the volatility of the Ibovespa. With regard to returns, the descriptive statistics are presented in the last three columns (R\_Ibovespa (R\$), R\_Ibovespa (US\$) and R\_CRB). Thus, the R\_Ibovespa represents the composite return of the Ibovespa, calculated using the difference in the logarithm of the Ibovespa. R\_CRB was calculated likewise. In the return CRB series, it was also possible to verify the excess kurtosis typical of financial series returns.

As regards the coefficient of variation, which measures the standard deviation relative to the average, it is observed that the return of the Ibovespa in dollars has the highest coefficient of variation. In other words, the Ibovespa dollar series was more volatile than the CRB. This can be observed in the minimum and maximum values. The Ibovespa had dollars in the period with a maximum daily drop of 18.24% and a high daily maximum of 19.61%. In relation to the CRB (R\_CRB) return, the largest daily drop was 5% and the largest increase in the period of the sample was 4.4%. Regarding the shape of the distribution, it can be noted from the Jarque-Bera statistic that it is possible to reject the hypothesis of normality in all cases, which was also to be expected in view of the presence of strong volatility in the variables, which generates leptokurtic distributions.

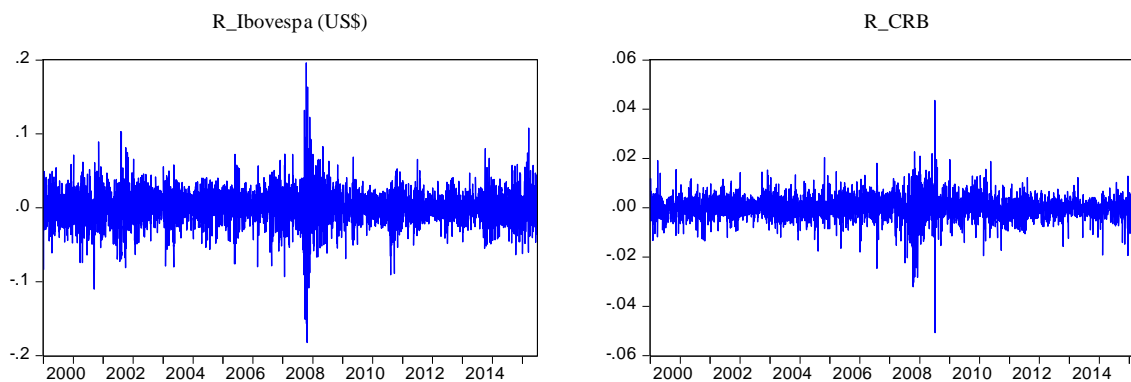
**Figure 1.** Ibovespa (R\$ and \$) and CRB (base index 100 = 1960) in the period January 3, 2000–June 30, 2016



**Source:** Elaborated from data obtained from Economática and Bloomberg.

Besides descriptive statistics, we recommend observing the plot of the data in the graphs. Thus, it can be seen in Figure 1 that the behavior over time of the Ibovespa and CRB indices is quite similar. In the initial sample period from 2000 to 2002, there was a fall, and since then both the Ibovespa and the CRB have experienced a growing trend, interrupted by the international financial crisis of 2008. After the crisis, there was a recovery, and the CRB exceeded the levels reached before the crisis, while the Ibovespa could not overtake it and entered a downward trend. The same downward trend can also be seen in relation to the CRB from 2011.

**Figure 2.** Compound return (Ibovespa in US\$ and CRB) in the period January 3, 2000–June 30, 2016



**Source:** Elaborated from data obtained from Economática and Bloomberg.

Following the graphs of the variables used in the research, the log differences are presented. The compound returns of both the CRB and the Ibovespa dollar demonstrate common features in the financial series, such as the presence of volatility clustering, as can be seen in Figure 2, which is compatible with distributions with heavy tails (leptokurtic), for which traditional OLS estimates result in waste with non-normal distribution.

Hereafter, the entire citation of the Ibovespa will refer to the stock index in dollars, which was used due to the compatibility in terms of monetary units with the CRB, as mentioned above. Figure 2 also shows that the international financial crisis, which originated in the US mortgage market, triggered a sharp increase in volatility in both the Ibovespa and the CRB, as the largest variations occurred in September 2008. These variations correspond precisely to the maximum and minimum values of the two series presented in the descriptive statistics in Table 1.

### 3.2 Econometric Model

The main econometric model used in this article is the GARCH multivariate model. Among the different possible specifications for the model, it was decided to use the BEKK specification, developed by Baba, Engle, Kraft and Kroner, which can be found in the study by Engle and Kroner (1995). The model description will be displayed in sequence, but it is noteworthy that, before estimating a GARCH multivariate model, it is necessary initially to estimate a vector autoregressive model (VAR model) and then to evaluate the cross-correlogram of squared residuals and cross-products of the residuals with the aim of identifying the possible presence of effects between variables.

Thus, initially an unrestricted vector autoregressive model was estimated according to the following specification:

$$y_t = A_1 y_{t-1} + \dots + A_N y_{t-N} + Bx_t + \varepsilon_t \quad (1)$$

where:

$y_t$  = vector of the endogenous variable

$x_t$  = vector of the exogenous variable

$A_1 + \dots + A_N$  and  $B$  = matrices of coefficients to be estimated

$\varepsilon_t$  = auto-correlated innovation vector

Prior to estimating the model, following Elliot, Rothenberg and Stock (1996), an ADF-GLS unit root test was applied to the original series and the return series composed of the CRB and Ibovespa variables in US\$. The ADF-GLS test is more potent than the recurrent form of testing used in the literature, such as the ADF, Phillips–Perron and KPSS tests. Based on the characteristics of the data, it is expected that the level series have a unit root and that the log difference is stationary. To estimate the VAR model, lagged choices of criteria are used. The choice of criteria is also based on the analysis of the correlogram with the goal of eliminating all autocorrelation and therefore generating waste with white noise. Even though the waste has white noise characteristics, the analysis of squared residuals and the cross-correlogram between the two variables may suggest some kind of effect on the conditional variance. In this particular

case, it is recommended to apply a multivariate GARCH model. The GARCH multivariate models were developed from the univariate models based on the work of Engle (1982), which introduced the Autoregressive Conditional Heteroskedastic model (ARCH model), and Bollerslev (1986), which extended the analysis to the autoregression model of Generalized Autoregressive Conditional Heteroscedasticity, which is referred to as the GARCH model. Subsequently, Engle, Granger and Kraft (1984) extended the analysis to bivariate models. In this sense, the evolution of univariate models into multivariate models can be seen based on the literature review by Bauwens, Laurent and Rombouts (2006).

Consider a series of multivariate returns  $\{y_t\}$ , where  $y_t$  is a dimension of univariate returns vector  $N \times 1$ . It is possible to write  $y_t$  in each instant of time, such as:

$$y_t = \mu_t(\theta) + e_t \quad (2)$$

where  $\mu_t$  is the conditional expectation of  $y_t$ ,  $\theta$  is a vector of finite parameters and:

$$e_t = H_t^{1/2}(\theta)z_t \quad (3)$$

where  $H_t^{1/2}(\theta)$  is a positive definite matrix  $N \times N$ . The vector  $z_t$  has an expected value of zero and variance equal to the  $I_N$  matrix, which is the identity matrix of order  $N$ . The term  $H_t^{1/2}$  represents a positive-definite matrix  $N \times N$ , which can be obtained by Cholesky decomposition. Additionally, it is possible to verify that  $H_t$  is the conditional variance matrix of the vector  $y_t$ .

Specifying  $H_t$ , which is the conditional variance matrix, is the object of study in the multivariate GARCH models. A general formulation of the model, developed by Bollerslev, Engle and Wooldridge (1988), states that each element  $H_t$  is a linear function of the lagged squared errors and errors of cross products with the lagged values of all the elements of the conditional covariance matrix, according to the following expression:

$$h_t = c + A \eta_{t-1} + G h_{t-1} \quad (4)$$

$$\text{where: } h_t = \text{vech}(H_t) \text{ e } \eta_t = \text{vech}(e_t e_t')$$

In Equation 4,  $A$  and  $G$  are parameters of the matrices of order  $(N + 1) N / 2$  and  $c$  is a parameter vector with  $(N + 1) N / 2 \times 1$ . Already the *vech* notation represents an operator that stacks the columns of the lower triangular part of an  $N \times N$  matrix with  $(N + 1) N / 2 \times 1$  vectors. The problem overparameterized is a characteristic of this embodiment, since, for example,  $N = 3$  estimates 78 parameters of the model. The adoption of some restrictions on the matrices  $A$  and  $G$  can reduce the number of estimated parameters. Accordingly, Bollerslev, Engle and Wooldridge (1988) proposed the diagonal VEC model (DVEC). In the DVEC model, it is assumed that the matrices  $A$  and  $G$  are diagonal and that each element of the  $h_{ij}$  conditional variance matrix depends only on its own lagged value and the previous values of the  $e_{it}$  and  $e_{jt}$ .

Ensuring that the conditional variance matrix is positive definite requires the model to be represented by applying the Hadamard product. In this type of operation, the product  $ij$  elements of two arrays of identical dimensions, generates a new matrix in which each element  $ij$  is the product of the  $ij$  elements of the original matrix. Thus, the model can be represented as:

$$H_t = C^o + A^o \odot (e_{t-1} e_{t-1}') + G^o \odot H_{t-1} \quad (5)$$

$C^o$ ,  $A^o$  and  $G^o$  are symmetrical  $N \times N$  matrices given by the relationship  $C = \text{diag}[\text{vech}(C^o)]$ ,  $A = \text{diag}[\text{vech}(A^o)]$  and  $G = \text{diag}[\text{vech}(G^o)]$ , and the notation  $\odot$  represents the Hadamard product. To estimate the variance and covariance, an exponentially weighted



moving average (EWMA) template may be used, which was initially applied to univariate models and then to multivariate VEC models in the following format:

$$h_t = (1 - \lambda)\eta_{t-1} + \lambda h_{t-1} \quad (6)$$

In the context of the DVEC model to estimate the variance and covariance for the EWMA, the positivity of  $H_t$  cannot be guaranteed without imposing significant restrictions on the parameters. The imposition of positivity was an evolution of the VEC model to the BEKK model developed by Engle and Kroner (1995). In this context, the BEKK model (1, 1, K) may be defined as:

$$H_t = C^* C^* + \sum_{k=1}^K A_k^* e_{t-1} e_{t-1}' A_k^* + \sum_{k=1}^K G_k^* H_{t-1} G_k^* \quad (7)$$

$C^*$ ,  $A_k^*$  and  $G_k^*$  are  $N \times N$  matrices wherein  $C^*$  is an upper triangular matrix. Unlike the VEC model, the parameters of the BEKK model have no relation to the values' passed volatilities or correlations between sets and thus constitute a special case of the VEC model. Restrictions to the identification of a BEKK model with  $K = 1$  are imposed on matrices  $A_{k,11}^*$  and  $G_{k,11}^*$ , which should be positive, in addition to the diagonal elements, which should also be positive. Aiming to reduce the number of parameters in the BEKK model, it is possible to apply a BEKK diagonal model, in which the matrices  $A_k^*$  and  $G_k^*$  are diagonal. It is a less general model than the DVEC model but ensures that the conditional variance matrix is positive definite. By reducing the number of parameters estimated by the model and due to the fact that it is one of the most used in the literature for contagion overflow volatility, the diagonal BEKK specification was the application selected for this study.

#### 4. Results and discussion

Initially unit root tests were performed on the original series and the CRB index and Ibovespa and on the returns series of compounds. As mentioned earlier, we performed the ADF-GLS test, as the ADF and KPSS tests, although used more intensively, have a lower degree of potency. In this context, Table 2 presents the results of the ADF-GLS unit root tests, which confirmed the presence of a unit root in the level of the series and the stationarity of the Ibovespa and the CRB in the logarithmic difference.

**Table 2.** Results of the ADF-GLS unit root tests

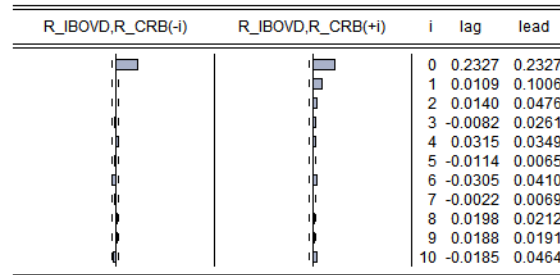
Variable	Lags	Constant	Trend	t-Statistic	Critical value 10%	Critical value 5%	Critical value 1%
CRB	4	Yes	Yes	-0.951701	-2.570000	-2.890000	-3.480000
$\Delta \text{LogCRB}$	9	Yes	no	-13.88098***	-1.616650	-1.940898	-2.565501
Ibovespa	1	Yes	Yes	-1.427209	-2.570000	-2.890000	-3.480000
$\Delta \text{LogIbovespa}$	7	Yes	no	-2.304293**	-1.616650	-1.940898	-2.565501

**Source:** Own work based on calculations made in the econometric package Eviews 9.0.

**Notes:** \*\*\* Rejection of the null hypothesis of the presence of a unit root at the level of 1%. \*\* Rejection of the null hypothesis of the presence of a unit root at the level of 5%. Critical values generated by the econometric package mentioned.

Considering the series of variable returns, it is possible, by means of a cross-correlogram, to assess any relationship between variables; the cross-correlogram between the Ibovespa and CRB returns, shown in Figure 3, indicates the presence of the structure in the conditional mean up to the lag. Therefore, we should initially model the vector returns through a VAR model.

**Figure 3.** Cross-correlogram between CRB and Ibovespa in US\$ in the period January 3, 2000–June 30, 2016



**Source:** Own work based on calculations made in the econometric package Eviews 9.0.

Given the existence of a relationship between the daily returns of the Ibovespa and those of the CRB, it is necessary to estimate a VAR paying special attention to the number of lags to be used in the estimate, which should eliminate the serial correlation of the residuals. Due to the characteristics of the CRB and Ibovespa series, for example daily, there is a serial correlation structure that must be eliminated carefully with the inclusion of a large number of lags. Thus, it was decided to conduct five tests using different criteria considering a model with 30 lags: a statistical test sequential LR modified with each error at the 5% significance level, the final prediction error test and the Akaike, Schwarz and Hannan–Quinn criteria. The results are shown in Table 3.

According to Table 3, there was disagreement between the selected criteria. While the Schwarz criterion pointed to the VAR (1) as the best model, the modified statistical sequential LR test selected the VAR (28) model as the most suitable. However, after estimating a VAR (1), which is a more parsimonious model according to the Schwarz criterion, we observed the presence of serial correlation in the waste, which in turn was eliminated only with the inclusion of 28 estimated lags. Therefore, the VAR (28) model, as indicated by the LR criterion, was selected as the appropriate model.

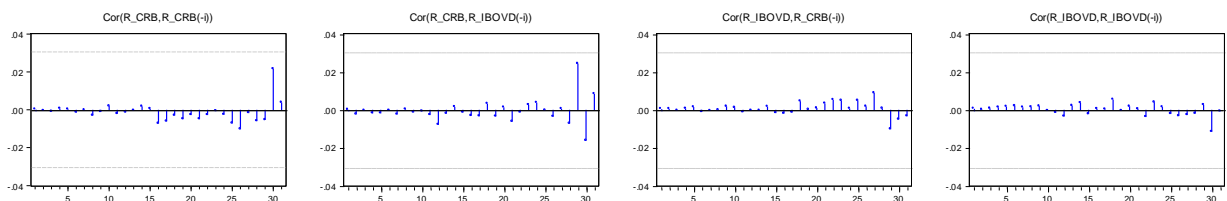
**Table 3.** Lag selection criteria for the VAR model

Defasagens	LR	FPE	AIC	SC	HQ
0	NA	1.07e-08	-12.67443	-12.67145	-12.67337
1	63.92256	1.06e-08	-12.68753	-12.67859*	-12.68437
2	26.78766	1.05e-08	-12.69193	-12.67704	-12.68667
3	7.093238	1.05e-08	-12.69172	-12.67088	-12.68436
4	38.99036	1.05e-08	-12.69900	-12.67220	-12.68953*
5	7.891932	1.05e-08	-12.69898	-12.66622	-12.68740
6	15.33477	1.05e-08	-12.70070	-12.66200	-12.68703
7	1.984016	1.05e-08	-12.69930	-12.65463	-12.68352
8	9.784169	1.05e-08	-12.69972	-12.64911	-12.68184
9	8.212049	1.05e-08	-12.69978	-12.64321	-12.67980
10	26.12705	1.04e-08	-12.70406	-12.64153	-12.68197
11	14.09305	1.04e-08	-12.70550	-12.63702	-12.68131
12	11.25513	1.04e-08	-12.70628	-12.63184	-12.67998
13	5.423717	1.04e-08	-12.70568	-12.62529	-12.67728
14	7.874101	1.04e-08	-12.70566	-12.61932	-12.67516
15	14.64633	1.04e-08	-12.70725	-12.61494	-12.67464
16	14.14602	1.04e-08	-12.70871	-12.61045	-12.67400
17	16.61495	1.03e-08	-12.71076	-12.60655	-12.67394
18	17.63489	1.03e-08*	-12.71305*	-12.60288	-12.67413
19	6.422922	1.03e-08	-12.71269	-12.59657	-12.67167
20	5.550832	1.03e-08	-12.71213	-12.59006	-12.66901
21	6.754916	1.03e-08	-12.71186	-12.58383	-12.66663
22	11.17386	1.03e-08	-12.71263	-12.57864	-12.66529
23	5.541266	1.03e-08	-12.71207	-12.57213	-12.66263
24	4.521762	1.03e-08	-12.71127	-12.56537	-12.65972
25	0.992800	1.04e-08	-12.70963	-12.55778	-12.65598
26	8.050122	1.04e-08	-12.70966	-12.55186	-12.65391
27	5.854069	1.04e-08	-12.70918	-12.54542	-12.65133
28	11.20804*	1.04e-08	-12.70997	-12.54025	-12.65001
29	3.410191	1.04e-08	-12.70890	-12.53323	-12.64684
30	7.315327	1.04e-08	-12.70877	-12.52714	-12.64460

Source: own based on calculations made in econometric package Eviews 9.0.

It should be noted that, despite the high number of lags, no overparameterized model problem exists, given that the data are daily and make up a sample with 4301 observations. In the VAR (28) model it was estimated 56 parameters and a constant. Thus, despite the high number of parameters, the estimate was ensured by presenting a number of observations that is more than 70 times the number of parameters. Figure 4 shows the model whereby effectively the VAR (28) was able to eliminate the presence of serial correlation in the residues of each series and in the cross-correlogram of waste.

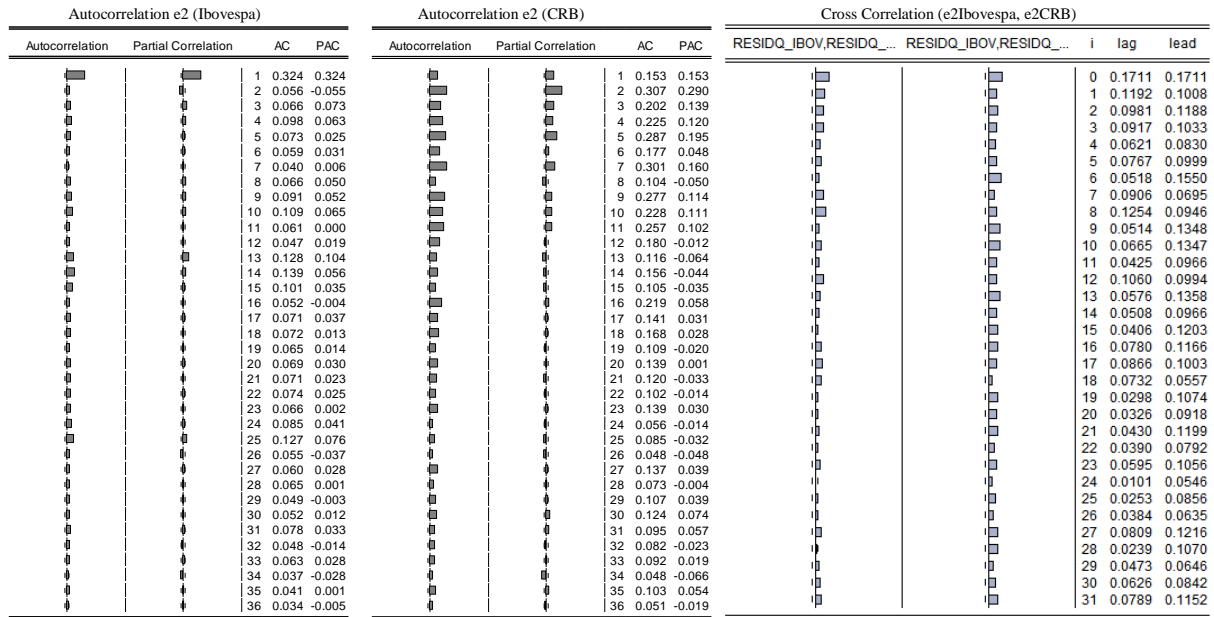
**Figure 4.** Residual correlogram and residual cross-correlogram between the Ibovespa in US\$ and CRB in the period January 3, 2000–June 30, 2016



Source: Econometric package Eviews 9.0.

In the sequence the autocorrelation function of squared residuals of the two VAR estimated model equations was generated. Additionally, the cross-correlation function of the square of the waste VAR equations (28) was estimated. Both were in accordance with the autocorrelation functions of squared residuals as compared with the cross-correlation of the squares of the residuals of the VAR model estimated; Figure 5 shows that there is a structure in the squares of the residuals and the conditional variance of the variables.

**Figure 5.** Squared-residual autocorrelations and squared-residual cross correlation between the Ibovespa in US\$ and the CRB in the period January 3, 2000–June 30, 2016



Source: econometric package Eviews 9.0.

Given the evidence presented in the cross-correlogram, we estimated a BEKK diagonal model, demonstrated by Engle and Kroner (1995) as cited above. The estimated coefficients with the respective standard errors are shown in Table 4. It can be seen that all the coefficients are significant. Table 4 further presents, in the second column, the reference coefficient for the estimated model of the BEKK matrix.

**Table 4.** Model results for the estimated BEKK

Equation coefficients of variance	Matrix	Coefficients	Standard error	z-Statistic	Prob.
C (3)	M (1,1)	9.24E-06	1.33E-06	6.969161	0.0000
C (4)	M (1,2)	2.94E-07	7.13E-08	4.119466	0.0000
C (5)	M (2,2)	1.36E-07	1.64E-08	8.302535	0.0000
C (6)	A1 (1.1)	0.257344	0.009819	26.21013	0.0000
C (7)	A1 (2.2)	0.151188	0.005358	28.21479	0.0000
C (8)	B1 (1.1)	0.956476	0.003341	286.2964	0.0000
C (9)	B1 (2.2)	0.984903	0.001018	967.0815	0.0000

Source: Econometric package Eviews 9.0.

Notes:  $GARCH = M + A1 * e_{t-1} * e'_{t-1} * A1 + B1 * GARCH_{t-1} * B1$ , wherein M is an undefined matrix and A1 and B1 are diagonal matrices.

$GARCH_{IBOV\_D} = M(1,1) + A1(1,1)^2 * (e_{IBOV\_D,t-1})^2 + B1(1,1)^2 * GARCH_{IBOV\_D,t-1}$

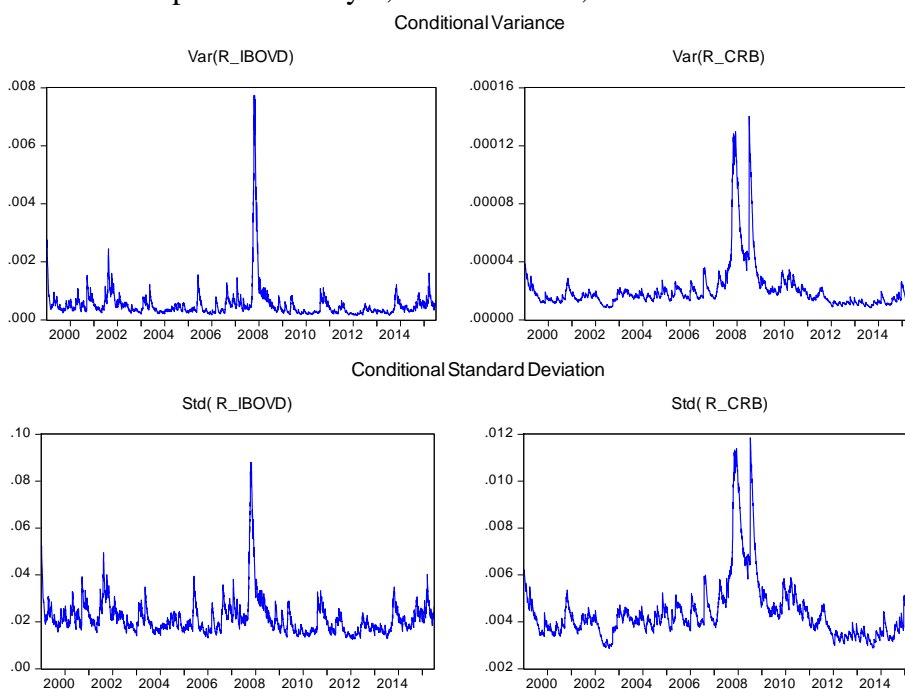
$GARCH_{CRB} = M(2,2) + A1(2,2)^2 * (e_{CRB,t-1})^2 + B1(2,2)^2 * GARCH_{CRB,t-1}$

$COV(IBOV\_D, CRB) = M(1,2) + A1(1,1) * A1(2,2) * e_{IBOV\_D,t-1} * e_{CRB,t-1} + B1(1,1) * B1(2,2) * COV(IBOV\_D, CRB)_{t-1}$

In general, the analysis of volatility series and volatility spillovers (contagion effect) in the context of the BEKK model is performed using the behavior of the conditional variance, the standard deviation conditional covariance and especially the conditional correlation. Note that, unlike other specifications, in the context of the BEKK model, the model parameters cannot be interpreted in the light of past values of the volatilities or correlations between sets. In this regard, Figure 6 shows, in the left graphs, the conditional variance and standard deviation of the Ibovespa and, in the right graphs, the CRB index. Increased volatility is apparent in several periods of the series, but the increase in volatility is very significant especially in 2008, both in the Ibovespa series and in the CRB series. As expected in times of crisis, there is a significant increase in the volatility of stock prices and commodity prices.

In the context of the BEKK model, the analysis of the conditional covariance and conditional correlation between two or more assets effectively allows the evaluation of the contagion effect. In this sense, Figure 7 shows the significant increase in the conditional covariance between the Ibovespa and the commodity prices in the period of the subprime crisis. Some signs of increased covariance in 2007 can be identified, but this effect is more significant in the last quarter of 2008, as shown in the graph on the left of Figure 7.

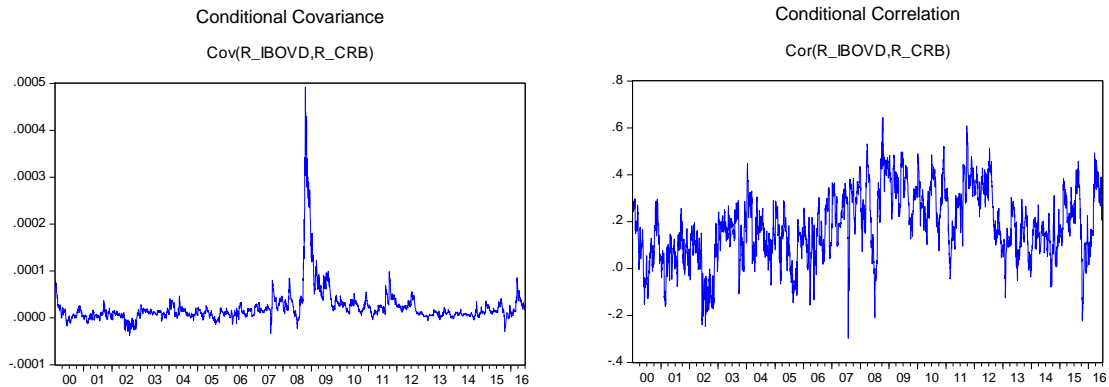
**Figure 6.** Conditional standard deviation and conditional variance of the Ibovespa returns in US\$ and the CRB in the period January 3, 2000–June 30, 2016



**Source:** Econometric package Eviews 9.0.

Additionally, in the right graph, it is possible to observe the behavior of the conditional time-variant correlation. The results indicate weak correlation between the conditional Ibovespa and the price of commodities in the analysis period, as can be seen in Figure 7. Although the height of the conditional correlation crisis has risen to about 0.60, which is expected in a situation of increasing market volatility, the conditional average correlation of 0.20 in the periods before and after the crisis suggests a weak relationship between the two variables, which allows diversification of investment in commodities and the Brazilian stock index.

**Figure 7.** Covariance and conditional correlation (CRB and Ibovespa) in the period January 3, 2000–June 30, 2016



Source: Econometric package Eviews 9.0.

In addition, the study confirms the contagious effect according to the definition Forbes and Rigobon (2002), causing a significant increase in the conditional correlation in the period of the subprime crisis. Thus, despite the weak relationship expressed by the conditional correlation throughout the period, the expansion of connections during the subprime crisis suggests the spillover of volatility.

Despite the existence of the contagion effect, which has also been identified in other studies evaluating the effects of oil prices on the stock exchanges of countries that export fossil fuel, such as Arouri, Lahiani and Nguyen (2011) and Teulon and Guesmi (2014), there is a weak correlation between the conditional and the CRB index of the Ibovespa, which ensures investors the possibility of diversifying between the two markets. The weak conditional correlation or weak relationship of interdependence observed during periods of absence of crisis suggest that it is possible to diversify risks between the Ibovespa and the commodities, despite the historical evolution of the stock index associated with companies operating in the sector, but that lost considerable participation in the last decades due to the very dynamic evolution of the companies and sectors that started to compose the Brazilian stock index.

## 5. Concluding Remarks

This article aimed to analyze the effect of contagion or volatility spillover between the Ibovespa index and the CRB index, which is the price of commodities in the international market. The approach is justified, inter alia, by the assessment of risk diversification and the perspective of investors, both active in terms of resource allocation. The importance of the analysis is partly due to the strong relationship presented by the Ibovespa, both in historical terms and in relation to the characteristics of the Brazilian economy and the commodity sector, in view of the considerable weight of commodity companies in the index. It should be noted that, in the recent period, the diversification of the companies and sectors in the composition of the main Brazilian stock index has increased. To achieve the goals, a GARCH multivariate model was applied for the primary purpose of identifying the contagion effect and conditional correlation, which corresponds to the degree of time-varying correlations between the Ibovespa and the commodity prices. Regarding the choice of the GARCH multivariate model, the diagonal BEKK specification was selected due to the possibility to estimate a small number of parameters and the fact that it has been widely used in the literature.

The research partially supported the hypothesis initially formulated. The analysis of conditional covariance clearly showed the contagion effect between the commodity prices and the Ibovespa; after the subprime crisis, we observed a strong increase in the conditional

covariance between the two variables. Similarly, we observed that the conditional correlation widened during the crisis, reinforcing the thesis of the presence of the contagion effect. However, the analysis of the behavior of the Ibovespa together with the commodity prices showed that the conditional correlation between the variables was relatively weak, reaching approximately 0.2 in the periods before and after the crisis of 2008, suggesting that investment in concomitant commodities and the Ibovespa is a risk diversification strategy, contrary to what is commonly supposed. The results, in terms of the contagion effect, are similar to those found by other authors, such as Arouri, Lahiani and Nguyen (2011) and Teulon and Guesmi (2014), who evaluated the effects of the transmission of volatility between oil prices and stock markets. On the other hand, and contrary to what was found in these studies, there was a weak conditional correlation between the commodity prices and the Ibovespa, which translates into diversification opportunities among stocks and commodities in periods of absence of crisis.

Note that, especially from 2010, there was a more intense gap in the trajectories of the Ibovespa index in Brazilian local currency and in dollars. In part, the most intense changes in the nominal exchange rate constituted an additional source of volatility for the Ibovespa, which may have influenced the conditional estimated correlation between the Ibovespa and the commodity prices in the international market. This fact is one of the limitations of the research. The estimated GARCH model with the Ibovespa in local currency could reduce the effect of the exchange rate, but it would make the two sets incompatible, as the CRB reflects the commodity prices in dollars. The second limitation is the fact that the CRB index reflects a basket of commodities in terms of diversification. It is much broader than the commodities related to companies listed on the Ibovespa index. Thus, the CRB index does not reflect accurately the commodity companies present in the Brazilian stock index. However, there is no availability of information on a daily basis that considers the commodities included in the Brazilian economy. To solve the issue of the incompatibility of the commodities present in the CRB and the Ibovespa, it is possible to estimate the presence of the contagion effect individually from changes in the price of oil on the shares of companies linked to the sector and to assess the conditional correlation estimated additionally. This can also be performed in relation to iron ore and other commodities, with specific results in terms of risk diversification. Finally, the study could be expanded with the application of different specifications of multivariate GARCH models to compare the results of the estimates. Given the importance of the issues, such possibilities should be included as topics on the future research agenda.

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