Fractal Analysis of Urban Forms: Study of Fractal Dimension and Municipal Human Development Index (MHDI)

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Abstract. The concepts of fractal geometry, which were developed by Mandelbrot, complement Euclidean geometry as they provide theoretical dimensional fundamentals for shapes whose irregularities cannot be properly interpreted by classical topological definitions. Urban studies supported by geotechnologies have great potential for applying Mandelbrot's theories as a conceptual reference for analyzing phenomena presenting fractal behaviors. In this approach, the fractal dimension of the territorial space occupied by cities is considered an indicator for understanding occupation patterns as one of the factors to be considered in urban planning policies. This study aims to apply these concepts using the box-counting method to calculate the fractal dimension of urban forms from eight Brazilian state capitals by using the Municipal Human Development Index (MHDI) as the selection criteria to find possible relationships that may support future studies and urban planning.

Keywords: fractal geometry, urban studies, fractal dimension, urban planning policies, box-counting method, urban forms, Municipal Human Development Index.

1 Introduction

The "State of World Population Report – 2011" published by the *United Nations Fund for Population Activities* (UNFPA) [1] noted that the global population has reached the seven billion mark. Of this, one-third lives in cities, and this proportion is expected to reach two-thirds by 2050.

This increase in the city-dwelling population represents one of the major change agents in urban areas, contributing to the configuration of complex patterns of space appropriation. Understanding the dynamics of this process is one of the challenges for scholars and researchers who analyze expansion phenomena and trends in the occupation of urban sites, aiming, for example, to propose more suitable projects for investment distribution and infrastructure development.

Fractal geometry, which was developed by the mathematician Mandelbrot in the second half of the last century, offers alternative approaches to analyze complex and irregular shapes. Batty and Longley [2], Frankhauser [3], Benguigui et al. [4], and others have developed urban studies using a fractal approach, arguing that the fractal dimension may be an important indicator for understanding the occupation of the territorial space of cities by their urban sprawl. Marques and Ferreira [5] suggested that the fractal dimension reveals the distribution pattern whereby the urban form evolves on the territorial space, and it may therefore be used "as a support to urban planning, proposing solutions to more homogeneous spatial expansion" [5].

This study proposes to use a fractal approach to analyze the distribution pattern of the urban forms from eight Brazilian state capitals.

2 Fractal Geometry

The term *fractal* — from the Latin *fractus*, corresponding to the verb *frangere* that means "to break" [6] — was coined by Mandelbrot to describe a class of objects and shapes whose irregularities cannot be properly interpreted by traditional Euclidean geometry.

Fractals also have features such as infinite details and self-similarity. Clouds, mountains, plants (Fig. 1), and animal organs are examples in nature that reveal fractal structures and present statistical self-similarity, i.e., the degree of irregularity is the same at all scales, but the "small pieces" are not exact copies of "the whole."

Mathematicians such as Cantor (1883), Peano (1890), and Von Koch (1904) built geometrical structures that were considered "pathological" and "mathematical monsters" by the scientific community because they presented non-intuitive properties [7]. Mandelbrot later recognized these properties as the same ones found in fractals in nature. The Cantor set (Fig. 2) has an easy building rule: it starts with a straight line, followed by a sequence of exclusion operations from the middle third of its previous parts [8]. In the limit, it represents a set of infinite disconnected points (Euclidian zero-dimensional) that are, however, limited to the finite length of the original straight line (Euclidian one-dimensional). The Cantor set is also an example of strict self-similarity, in which the parts are exact copies of the whole.

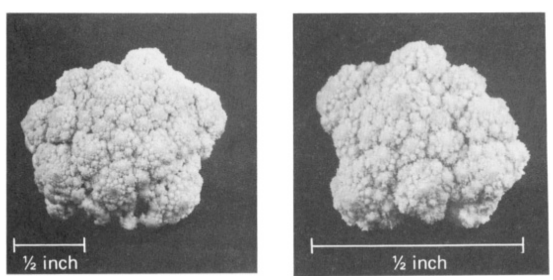

Fig. 1. Fractals in nature. A cauliflower is an example of statistical self-similarity. The small pieces are variations of the whole [9].

The Euclidian topological dimension of an object is defined as an integer value that represents the independent coordinates required to describe each of its points [9].

Fractal geometry allows fractional values for a dimension to define an object or shape with dimensional characteristics transiting between the Euclidian integer dimensions, such as the Cantor set, the fractal dimension of which is ~0.6309 [6].

Fig. 2. Cantor set (Cantor, 1883).

2.1 Box-Counting Method

The box-counting method is a special form of Mandelbrot's fractal dimension, with great applicability to shapes in the plane [9]. This method involves covering a structure or shape with a grid of boxes with side length *s* (Fig. 3). The number of boxes that intersect the structure is *N*, obtaining *N(s)*. Then, repeat this process with smaller boxes to obtain additional *N(s)*. Finally, the logarithm points log(N(s)) are plotted versus log(1/s) and a straight line is fitted along them. The slope of this line gives the fractal dimension [9].

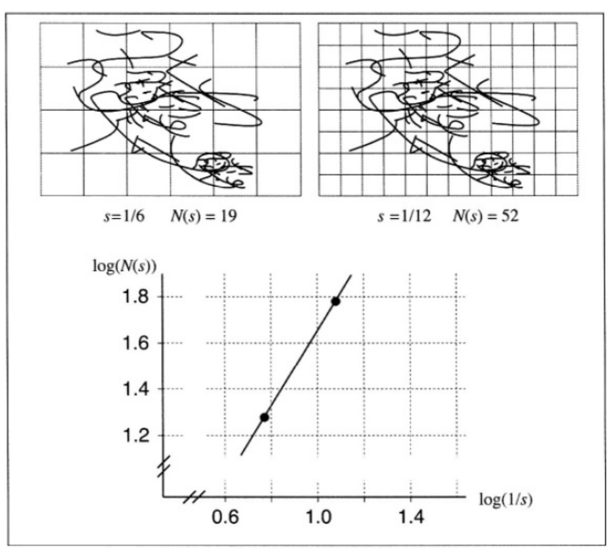

Fig. 3. The box-counting dimension. An example with two grids. The slope of the line is log(52/19)/log(2) ≈ 1.45 [9].

3 Fractal Dimension and MHDI

The *United Nations Development Program* (UNDP) in conjunction with the *Applied Economics Research Institute* (IPEA) and the *João Pinheiro Foundation* have published the “Atlas of the Human Development in Brazil – 2013” with a list of the Human Development Index (HDI) of Brazilian cities, called as the Municipal Human Development Index (MHDI) [10][11].

Geographical information system (GIS) programs provide tools to analyze and manipulate satellite remote sensing images in order to identify urban areas to be researched. They can use the box-counting method to calculate the fractal dimension of urban areas from the Brazilian “top 4” and the last four state capital cities listed in the MHDI ranking.

This study analyzes and interprets these results to identify possible relationships or trends among these indicators, namely, fractal dimension and MHDI, that may support future studies and urban planning.

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